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Conversations in Development studies

Econophysics: Exploring a New Dimension to Economic Analysis & Reasoning

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Chitrakalpa Sen
Chaos and Complex System:
A Discussion

Sudip Patra
Quantum Modelling in
Human Decision Making: Current Status

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ABOUT CIDS

CIDS (Conversations in Development Studies) is a peer-reviewed, quarterly research publication produced by the research team of Centre for New Economics Studies, O.P. Jindal Global University. The student-led editorial publication features solicited research commentaries (in the range of 2500-3000 words) from scholars currently working in the cross-sectional aspects of development studies. Each published CIDS Issue, seeks to offer a comprehensive analysis on a specific theme identified within the scope of development scholarship.

The editorial team's vision is to let CIDS organically evolve as a space for cultivating creative ideas for young research scholars (within and outside the University) working dexterously to help us understand and broaden the development discourse through conceptual and methodological insights on contemporary issues.

Any research commentary submission shall feature: *a) brief review of the literature on a given research problem; b) the argument made by the author with details on the method used; c) documenting the findings and relevance of them in the larger scope of the literature;* and (in some instances) *d) present a brief policy action plan for agencies of the state* (to address the issue-highlighted in the commentary). There are no pre-identified limitations or restrictions to methodological frameworks used by scholars (writing the commentary). However, the method incorporated in any accepted submission must explain its relevance in context to the nature of the study and the cited literature review.

CONVERSATIONS IN DEVELOPMENT STUDIES (CIDS)

Volume 1: Issue 3 (Econophysics: Exploring a New Dimension to Economic Analysis & Reasoning)

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ABOUT THIS ISSUE

Econophysics: Exploring a New Dimension to Economic Analysis & Reasoning

"*Despite their very different ages, physics and economics have been developed and extended along the two sides of the same river for a long time. Crossing the river signifies the efforts made to connect the side of physics with the side of economics, or more generally, the side of the natural sciences and the side of the social sciences.*"

(Shu-Heng Chen and Sai Ping L, 2011)

For decades, scholars in field of natural and social sciences have been trying to bridge the gap across their respective scholarships with other disciplines. Over recent years, methodologies of research have come to a turning point where the global emergence of liberal arts and sciences have allowed space for natural sciences to become entwined with social sciences. There has been an upsurge in interest across disciplines, for example, Physics, Mathematics, Cognitive science, Psychology on one hand; and Economics, Political Science, Sociology as social sciences on the other hand.

As a case in point, there have been studies reflecting the influence of Physics on economics earlier too, for example, general equilibrium analysis has been deeply motivated by thermodynamics; even, the Black-Scholes model for asset pricing has been influenced by mathematical principles of thermodynamics. However, the extension of quantum theory to social sciences is exciting and ontologically different from the traditional approaches, since in quantum theory- randomness is true and does not stem from ignorance-as in standard Bayesian analysis. It goes without saying that in social science, we always grapple with degrees of radical uncertainties.

Econophysics, an interdisciplinary research field remains deeply associated with applying theories and ideas developed in physics to solve problems and uncertainties in the field of economics and, predict the behavior of financial markets. At a time when mainstream economics fails to see itself moving beyond

the neoclassical paradigm- in spite of how behavioral economics and network effects have demonstrated severe limitations of the standard theoretical paradigm- Econophysics and Quantum modeling describes and predicts real life behavior significantly better.

Let's take an example to understand this better. Physicists use *'Power Law'* to explain the relative change in one quantity leads to a relative change in the other quantity. Power Law is used to explain the behavior of gas molecules. Inverse cubic power law has been used by mathematical physicists over time to predict the financial markets, the up or down in large markets, stocks, bonds or currencies market. In this regard, physicists see income, wealth and financial assets as atoms and use their knowledge of science to predict the behavior in financial market. The derived mathematics from physics in economics has been successful in linking markets and natural phenomenon, say what effect the tsunami has on financial market. This has led to the discipline of Econophysics- also popularly known as '*Physics of finance'*.

In financial markets, it is hard to track the movement of each and every stock simultaneously. Hence, concept of entropy can be used to explain the performance of portfolio and market. Financial markets are dynamic and subjective to collective decision-making. The science aims to explain the decision making since logic we follow is not simple and the standard theory cannot explain the complications. However, quantum physics deal with the behavior of material at quantum level the same logic is utilized in the field of Econophysics to explain the movements in market. Financial markets are laden with bubble and busts which are explained by different beliefs. The divergence of these beliefs can be captured by the vision of quantum mechanics.

"Few if any economists seem to have realized the possibilities that such invariants hold for the future of our science. In particular, nobody seems to have realized that the hunt for, and the interpretation of, invariants of this type might lay the foundations for an entirely novel type of theory."

Schumpeter (1949, p. 155), while discussing the Pareto law

Quantum decision theory (QDT) is often used by the decision makers to give a quantitative prediction in the portfolios, a decision to choose between a risky and a certain lottery, both in domain of the gains concerning the average effect of subjectivity of people's decisions. Quantum decision theory considers the probabilistic and set theory to reach a decision of choosing the assets given the gain and risk levels.

Classical decision making based on the utility maximization approach is seen to be failing in the complex cases where the level of risk and uncertainty is relatively high. QDT is an answer to this failure of classical approach. Thus, we see here a

combination of the game theory of economics and quantum theory of physics to decide about the lotteries associated with gains' probability and risks levels. The field if often referred to as 'Quantum Economics'.

This Journal Issue seeks to make the field of Econophysics more widely known and accessible among scholars outside the discipline. The aim- with the two research commentaries published here- is to provide an insight into the working of financial markets and decision-making in economics. Risk is something to be understood and econo-physicists are working towards exploring the width and depth of it.

Major revolutions in sciences includes Relativity Theory, Quantum Mechanics and the Chaos Theory. The first paper delves into Quantum Mechanics. In the paper titled "Quantum modelling in human decision making: *Current status"* by Professor Sudip Patra. there is a paradigmatic shift in decision theory and social sciences have been tracked from late 90s to the current status. Human decision-making behavior is a highly complex and confounding process which many have tried to explain but only to certain limits. Accordingly, efforts in the field of Econophysics include the application of quantum modelling, derived from quantum physics, to understand and explain human cognition. The development in this field are discussed in the paper along with their application to specific human traits.

The second paper titled "*Chaos and Complex System; A Discussion",* by Dr. Chitrakalpa Sen forges into the application of '*Chaos theory'* in the field of Econophysics. It explains the characteristics of a chaotic system in the sense that it deals with non-linear behavior of a system and how these can be utilized for the effective application of the theory to understand economic phenomenon. The paper also delves into the relation or lack thereof between chaos and complexity.

The journal succinctly attempts to display some of the recent development and research in the specialized field of Econophysics with emphasis on two physical theories used therein, namely' *Quantum modelling'* and '*Chaos theory'*. Our objective here is to introduce these different areas in an endeavor to explain market changes and theorizing risk-management is an essential tool in developmental studies. While editing submissions, we do realize for a greater need of development in the field of Econophysics, particularly in its correspondence with economists and social scientists, who are less aware about the applications of Quantum mechanics (and Econophysics).

Chaos and Complex System: A Discussion

Chitrakalpa Sen[1]

Abstract

Economics is a complex system which is continuously evolving through interactions among the agents. The three most important revolutions in the field of science in twentieth century have been the theory of relativity, the theory of quantum mechanics and the chaos theory. The first two theories rely a lot upon calculus, which stands on the premise of simple linear approximations of nonlinear behavior. But chaos theory is different from the first two in the sense it deals with non-linear behavior of a system. For a system to be chaotic, it must be nonlinear, deterministic and sensitive to initial condition. Additionally, a chaotic system is essentially deterministic rather than stochastic, although deterministic and stochastic sound completely opposite to each other.

Keywords: Chaos, Complex, Calculus, Deterministic, Stochastic, Reductionism

1. Flutter by, Butterfly[2]

The recent Netflix film "*Black Mirror: Bandersnatch*" (2019) offers its viewers with a unique experience. The viewer gets to make decisions for the film's protagonist, Stephan and depending on the decision, the consequences vary from similar to wildly different. Although for movie buffs this is a unique

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[1].Dr. Chitrakalpa Sen is an Assistant Professor (Economics) at Jindal Global Business School. Sen teaches courses in the areas of Economics and Econometrics. His primary area of research is the study of high frequency financial market dynamics. Sen's other research interests include but not limited to Econometrics of green stocks and models of shared economy. His papers are regularly published in international journals. He has authored/co-authored a number of books with reputed international publishers.

[2] *Flutter by, Butterfly* by Josh Woodward, *The Simple Life*, Snooter Records, 2008.

viewing experience, this is a common phenomenon in real life. Our decisions, some of which may be apparently unimportant may lead to significantly different events. Our current state is necessarily a function of our cumulative past decisions. In fact, for everyone who is reading this article now, all your decisions you have ever taken in your life has culminated into your reading this line on this day at this very moment. This is *sensitive dependence to initial conditions*, an important characteristic of a nonlinear system.

One common example of *sensitive dependence to initial conditions* or simply *SDIC* widely given is that fluttering of a butterfly's wing in Brazil may lead to a tornado in Texas (Lorenz, 1972). Although somewhat exaggerated, it conveys the general idea of sensitive dependence. In popular culture, this is known as *The Butterfly Effect*. The chaotic butterfly took its first flights in a meteorology lab at MIT in 1961. Professor Edward N. Lorenz was conducting a weather simulation in his lab that had twelve variables. However, in those pre-computer days when one had to feed the data manually every time, Professor Lorenz continued with his earlier results after a break but eventually truncated the values from six decimal places to three. An apparently minor change resulted in a drastically different simulated weather pattern. This incidence led Lorenz to the idea that small changes can have big consequences and because knowing the initial conditions to infinitesimal precision is realistically impossible, long run forecast in a nonlinear system is nearly impossible[3].

This is the basic characteristic of a chaotic system, a continuous movement from order to disorder and back.

2. Unreasonable ineffectiveness of Mathematics[4]

The three most important revolutions in the field of science in twentieth century have been the theory of relativity, the theory of quantum mechanics and the chaos theory. The first two theories rely a lot upon calculus, which stands on the premise of simple linear approximations of nonlinear behavior. But chaos theory is different from the first two in the sense it deals with non-linear behavior of a system (Sarkar A., Chakrabarti G., Sen C. 2013).

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[3] An interesting example of the butterfly effect can be found in the old poem "*For the want of a nail the shoe was lost/ For the want of a shoe the horse was lost / For the want of a horse the rider was lost / For the want of a rider the battle was lost / For the want of a battle the kingdom was lost / And all for the want of a horseshoe-nail.*"

[4] Eugene Wigner, "*The Unreasonable Effectiveness of Mathematics in the Natural Sciences*," Communications in Pure and Applied Mathematics, vol. 13, No. I (February 1960). New York: John Wiley & Sons, Inc.

The origin of Chaos Theory lies in 1887's Sweden, when the French mathematician Henri Poincaré attempted to solve the 3-body problem, a simplified version of the *n*-body problem. Poincaré's work was a response to a mathematical competition held by King Oscar II with an aim to understand the dynamics of the planetary motion in the solar system. Poincaré identified that with the slightest change in the initial position of the three bodies (although he didn't call it sensitive dependence, the idea was precisely the same), the trajectory or the orbits travelled by those will change drastically. According to him, "*It may happen that small differences in the initial positions may lead to enormous differences in the final phenomena. Prediction becomes impossible*". This was a significant development from the perfectly predictable Newtonian systems. Poincaré had just discovered the context of *deterministic chaos*. In a system characterized by deterministic chaos the future behavior is guided by the initial conditions. So, if the initial conditions are known with infinitesimal precision, the future states can be predicted precisely. However, perfect knowledge of the initial conditions would require a virtually massive amount of data which makes perfect prediction realistically impossible. Therefore, the future states, although seemingly random (for a lack of predictability), has a hidden order (for theoretically, it can be predicted if the perfect and precise initial conditions are known). This is the hidden *Order in Chaos*. Although Poincaré was the father of Chaos theory, the name wasn't coined until Tien-Yien Li and James Yorke introduced the term "chaos theory" in their 1975 paper "*Period Three Implies Chaos*".

3. Big Fleas, Little Fleas[5]

The Newtonian principle of *reductionism* stands on the idea that a set of complex phenomena can be broken down and explained as an aggregation and interaction of simpler parts. The principles of *reductionism* was explained succinctly by René Descartes in "Discourse on the Method of Rightly Conducting the Reason" (1637) as*, "to divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution"* and "*to conduct my thoughts in such order that, by commencing with objects the simplest and easiest to know, I might ascend by little and little, and, as it were, step by step, to the knowledge of the more complex*".

Calculus is based on this premise. Simpler parts are solved to reach the solution of the bigger, more complex problem. However, there are complex systems, which when reduced to smaller sets remain as complex as the whole. Introduction of fractals (Mandelbrot, 1975) brought an end to reductionism.

[5] An interesting example of self-similarity - *Big fleas have little fleas/upon their backs to bite 'em /And the little fleas have lesser fleas,/And so ad infinitium.*

Fractals are geometric shapes which are self-similar, i.e. they don't become simpler when magnified to any level.

Self-similarity is abundant in nature. It is seen on the coastlines, leaves, vegetables and even terrains to name a few. If we keep zooming in on a coastline with the most powerful telescope from space, we'll get from a rugged coast to a rugged rock to a rugged pebble to the last rugged dust particle, making perfectly precise calculation of a coastline impossible. The smaller parts here don't get any simpler than the bigger ones. The terrain of Earth from space looks very similar to the terrain under our feet (except of course the manmade structures), complete with water, mountains and forests, at a much smaller scale. Self-similarity can also be seen on the leaves of fern or on a Romanesco broccoli (Figure 1 and Figure 2).

Figure 1. Fractals in nature: A Fern Leaf

Figure 2. Fractals in Nature: Romanesco Broccoli

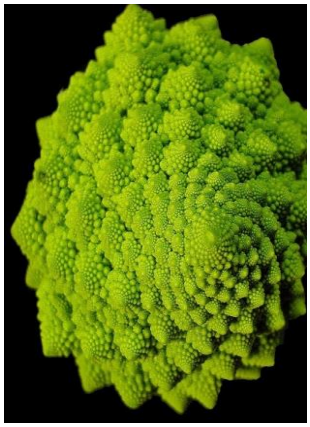

Figure 3. Mandelbrot

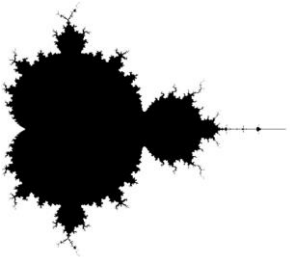

Image source:
Fig. 1. http://fractalfoundation.org/OFC/fernborg.jpg,
Fig. 2. https://i.ytimg.com/vi/9sVLMGm4Pgc/maxresdefault.jpg ,
Fig. 3. https://upload.wikimedia.org/wikipedia/en/e/ef/Mandelbrot_black_itr20.png

4. Understanding Chaos

For a system to be chaotic, it must be nonlinear, deterministic and sensitive to initial condition (Chakrabarti and Sen, 2012). Additionally, a chaotic system is essentially deterministic rather than stochastic, although *deterministic* and *stochastic* sound completely opposite to each other. The idea of determinism, which had its roots in the Newtonian principles of the 17th century was furthered in the 19th Century, by the French physicist Pierre Laplace. He wrote, "*We may regard the*

present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes." The above idea of a deterministic world, where the future states depend on the past is famously known as "*Laplace's Demon*", is based on the assumptions of: (a) a perfect knowledge of all the laws of nature working on the system, i.e. *all of the forces that animate nature* , (b) an ability to identify all possible states of the system i.e. *the mutual positions of the beings that compose it* and (c) infinitely vast and capable computational resources, *vast enough to submit the data to analysis* (Smith and Smith, 2007).

This sensitive dependence to initial conditions makes chaotic systems realistically impossible to predict unless perfectly precise data about the past states are available. While in a linear system, as iterations increase, the error remains proportionately same, in a nonlinear (and chaotic) system, the error increases proportionately and reaches beyond 100% after a small number of iterations, making any prediction impossible in the long run.

A chaotic system's evolves with time. The *state space* is the collection of all possible states in a dynamic system (Smith and Smith, 2007) and the path the dynamic system evolves in with time is known as the trajectory or orbit. A dynamic system tends to evolve towards a long term (*attracting*) set of numerical values in the state space known as attractors. A dynamic system settles down on an attractor in a long run. A chaotic system follows a *strange attractor* characterized by sensitive dependence to initial conditions and fractal structure, with unpredictable motions, which are globally stable but locally instable. On a strange attractor, two points initially infinitesimally close to each other (the nearest neighbors) would diverge from each other at an exponential rate over iterations, due to sensitivity to initial conditions (Ruelle and Takens, 1971), never leaving the attractor. On a Lorenz system, even points starting with very different initial conditions, tend to settle down on the attractor (in this case, a butterfly shaped strange attractor, a *Lorenz Attractor,* Figure 4.).

Signature of deterministic chaos can be detected in a system using the Lyapunov exponent, a number measuring the rate of divergence in the distance between nearest neighbors. Although the Lyapunov exponent in a dynamic system in a state space can take a range of values, the maximum Lyapunov exponent (MLE) is of most important for deterministic chaos. A positive MLE signifies definitive presence of chaos in the system.

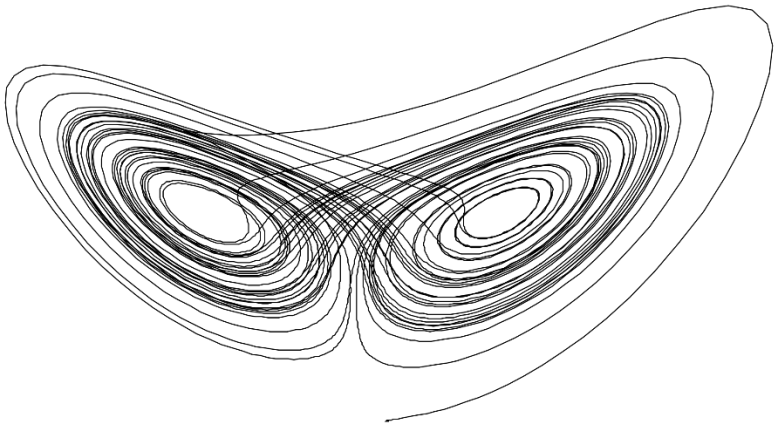

Figure 4. Lorenz (Strange) Attractor

5. Chaos vs. Complexity

Chaos and complex systems are often discussed together. Some claim that a chaotic system is a subset of a larger complex system. In order to understand the relationship (or the lack of it) between the two, this section takes a peek into a complex system.

It is important to understand that although in our daily uses we often use the terms "complex" and "complicated" interchangeably, there is a basic difference between a complex system and a complicated system. Preparing a five-course gourmet dinner is a complicated process, but not necessarily a complex process. A complicated system lends itself to reductionism. A chef can break down the preparation of the five-course gourmet dinner into smaller bits and correct execution of each one of the smaller parts would lead to an identical result. There is little unpredictability in a complicated system. A good restaurant would serve its customers the same dish again and again with very little variation, no matter how complicated the recipe is. A complex system on the other hand is unpredictable.

Complexity is necessarily a phenomenon that arises out of interaction among several constituent agents. A simplified example of a complex system is that of a traffic jam. Imagine a typical traffic jam, several kilometers long at a busy city intersection on a Friday evening. The car drivers have collectively decided to use that particular road at that particular time, with almost no central coordination or communication among them. This emergence of a collective behavior without any central coordination is one of the main characteristics of a complex system. This is also known as emergent phenomena. This phenomenon cannot be understood by observing a single agent in the system but is an outcome of

the interactions, the collective behavior. but the whole is more than the sum of individual parts. The emergent phenomena are visible all around us, from ant colonies to global weather patterns to human evolution to biological systems. Some examples of a complex system include governments, families, societies, ecological systems, evolution, human biology, the weather, economy, organizations, information systems and financial markets (Mathews et al, 1999; Pribram, 1996; Benbya & Mckelvey, 2006; Pascale et al, 2000). The field of complex systems covers different areas of natural, biological and social sciences and focuses on certain questions about parts, wholes and relationships (Gell-Mann, 1994, 1995; Prigogine & Nicolis, 1989; Arthur, 1995; Stacey et al, 2000; Adami et al, 2000; Bar-Yam, 2002). These questions are relevant to all traditional fields. Because of its interdisciplinary character, there is no single theory of complexity; rather, there exists a multitude of approaches arising out of the diverse disciplines that span this multifaceted field.

6. Understanding complexity

To understand what makes a system complex, it is first necessary to have an overview of such systems. Some of the characteristic features of a complex system are as follows (Latora & Marchiori, 2004; Baranger, 2001, Johnson, 2009).

- *A complex system contains many interacting constituents or agents*. These agents should be governed by nonlinear interaction. The complexity of the system arises entirely out of the interactions among large number of agents forming a network. More importantly, a complex system displays emergent behavior. In our simplified example of traffic jam, the collective decision of the drivers to take a particular road at a particular time creates the congestion. Take the 2008-09 financial crisis as another example. A single trader's risky practices might have relatively little impact on the financial market, but the emergent behavior literally brought the US economy on its knees.
- *The structure of a complex system spans several scales*. Take the government as an example; there are different levels (scales) such as local government and the state and the union government. Each scale is characterized by a clear structure.
- *A complex system exhibits* self-organization, which occurs when an emerging behavior changes the structure or creates a new structure. A complex system at one level is made of complex systems of lower levels, each continuously interacting with the other, creating a nested network.
- *A complex system is characterized by feedbacks*. An agent's current behavior is influenced by past events or present events taking place at another part

of the system the agent is a part of. For example, the financial market is characterized by phenomena such as long memory (past events influencing present outcomes) or spillovers (across markets). In our simplified traffic jam example, a congestion in a particular route may lead the drivers to take another route, causing congestion in the new route as well.

- *A complex system is typically open and alive*, i.e. it can be influenced by external environment and it evolves over time, often in a manner completely unpredictable due to the nonlinear interactions among the agents.

One must understand that complexity does not imply chaos, neither a chaotic system has to be complex, although both systems are dynamic, evolving over time (Bertuglia, Vaio, 2005). A chaotic system is essentially deterministic in nature. We usually associate the term "chaos" with disorder and randomness. However, if the initial conditions are known to infinitesimal precision even a chaotic system can be predicted with accuracy. On the other hand, a complex system is truly unpredictable.

7. Conclusion: Living on the razor's edge

The world around us is nonlinear. However, most of the tools used to explain this world are linear and that creates a problem. Take economic models for example. Most of the important economic models are built on the underlying assumption of linearity. This is one of the reasons most economic models do not work very well in real world. The economy is a complex system which is continuously evolving through interactions among the agents. The reason why often financial experts are unable to predict the market's behavior, often to disastrous consequences is because of this sensitive dependence to initial conditions. The financial market is a wonderful complex system, made of numerous agents, exhibiting tremendous self-organization and most importantly runs on a feedback loop. Take the stock market. The buying and selling behavior of traders depend on the market prices. The traders' expectations regarding the prices change the demand of the stocks and actually influence the prices, which in turn, will influence the expectations once again, thus feeding back information into itself, creating a feedback loop. Most financial markets, including the stock market are also deterministically chaotic, making it inherently unstable, balancing precariously on a razor's edge. The 2008-09 crisis is the biggest proof of that. For decades the way experts thought the market works, came crashing down. In words of Alan Greenspan in 2008, "*The whole intellectual edifice…collapsed in the summer of last year.*" The sensitive dependence is a particular handicap in trying to forecast in an economic system because of its inherent chaotic nature. Traditional linear analysis considers an equilibrium as the only possible solution. In a chaotic system, even a disequilibrium is equally likely as the system alternates

between a predictable, ordered behavior (such as a crash) and completely random behavior. Presence of deterministic chaos would cause future forecasts to fail by a large margin. That essentially implies that any long-term policy goals would be useless as the market (and the economy) continues balancing itself on the border of order and disorder.

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Quantum Modelling in Human Decision Making: *Current Status*

Sudip Patra[6]

Abstract

Since late 90s a new paradigmatic shift began in decision theory and social sciences as a whole. Due to the numerous shortcomings and limitations of the standard decision theory (based on Kolgomorovian measure theory, in effect Bayesian decision theory) scholars attempted to build a new decision theory based on the mathematical and logical foundations of Quantum theory. Last decade has witnessed a surge in Quantum-like modelling in decision theory with good success. Many so-called anomalies in human behaviour, viz, order effects, failure of sure thing principle, conjunction and disjunction effects have been explained to a great extent by this still emerging paradigm. However, there are still some challenges ahead for Quantum theoreticians. The current paper briefly summarises the main contributions and challenges or opportunities ahead of us who work in this emerging paradigm.

Key words: Quantum-like, Bayesian model, Hermitian operators, human cognition, Hilbert space, Order effects, Conjunction and Disjunction fallacies

1. INTRODUCTION

Failure or limitations of standard decision theory has been noted many times since the seminal works of cognitive scientists (Tversky and Kahnneman 1992). There has been a good response by the behavioural economics school (Thaler 1991) with various heuristics based and Bayesian learning models (Thaler 2000). However, there was always a lack of coherent and comprehensive theory which

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could resolve all imminent paradoxes, or anomalies, as already noted by theoretician's mentioned above. Quantum-like modelling in social science (Haven and Khrennikov 2013) designed since late 90s has come across as a brilliant candidate for the very purpose (Haven et al 2017). A new decision theory paradigm is on rise in a true sense.

It may surprise many that the mathematical and logical set up which was historically formulated for describing and predicting microphysical world (Haven et al 2017) can be so fruitful in the domain of human cognition. However, such is the nature of abstract mathematical modelling, one can recall the history of calculus which too was invented for describing physical systems.

Scholars find that there is a critical gap between the standard Neoclassical theoretic predictions and the real-life data driven results, such anomalies are profound, in the areas of decision making applied to financial markets, game theory, behavioural experiments, mathematical psychology, and allied areas. The anomalies which have been found between the standard utility maximization theory and the real life data, have been classified under various categories, such as, failure of the sure thing principle (which is a central premise of the expected utility theory, EUT), presence of disjunction effects in decision making, presence of order effects in decision making, and overall the failure of the basic law of total probability in decision making under the contexts of ambiguity and uncertainty.

Again, the failure of EUT to predict such behaviors also indicates the failure of classical probability theory/ classical Boolian logic theory in predicting human behavior in such contexts. The reason being that the standard EUT upon which the decision theory/ the standard Neoclassical paradigm is again based on the classical set theory of probability. Kolgomorov (1933) provided the modern foundation of the measure theory of probability, which is deeply embedded in the Boolian logic theory. Again, the later developments in the line of Bayesian probability updating theory/ Bayesian learning theory is also deeply embedded in the classical probability theory only. For example, the law of total probability which is central to the Bayesian paradigm is basically derived from the set theory of probability.

Given this backdrop, QDT or quantum decision theory, has emerged as a strong alternative for decision making theory in general. Again, the reason for QDT 's success in explaining the paradoxes (as mentioned above) is based on the fact that QDT is essentially a non-classical logic based theory. The probability measure in QDT is based on the Hilbert space formulation, which is in general an infinite dimensional complex space (in social sciences however we can work with the finite dimensional real subspace of the total Hilbert space), and the measurement of probability is based on the projector measure (either standard projection operators as used in Quantum Mechanics or more general Positive

Operator Value Measure operators which are more suitable for specific scenarios in decision making contexts, for example un-sharp measurements) which collapses (as in the standard Copenhagen version of QM) the general superposed mental state / belief state of the agents into Eigen subspaces of the Hilbert space, which then becomes the final state of belief.

Hence QDT helps us in one, measuring the probability of choosing a specific decision state, and two, helps updating the belief state of the decision maker accordingly. There are alternative mathematical formulations available for such measurements (please refer to, Quantum Social Science 2013). the striking success of QDT is evident from the significantly better match of the predictions with the real data points, in various scenarios, from prisoner's dilemma experiments to asset pricing theory to general probability updating. In probability updating too, QDT can avoid some central problems which Bayesian probability updating face, namely, the zero prior updating problem, hence this theory can be used in special scenarios where agents do update their posteriors to high values given that prior probabilities are near zero/ non-informative, which is impossible in Bayesian paradigm (Haven and Khrennikov 2013).

1. SPECIFIC TRAITS IN HUMAN DECISION MAKING

FAILURE OF SURE THING PRINCIPLE

In the standard decision theory, and in standard game theory irrelevance of irrelevant alternatives principle is well known. In a prisoner's dilemma condition (Rasmunsen 2012) dominant strategy equilibrium does suggest that irrespective of what the other player choose the other player should always choose to defect/ not cooperate, non-cooperation being the dominant strategy. However real people do behave drastically differently when the same game is played in the context of uncertainty / ambiguity, for example when the players have no idea about the move of the other player. Haven and Khrennikov (2013) have listed many such deviant behaviour of players, which cannot be described by the standard decision theory mathematics. However, the same authors, and many others (Haven et al, 2017), have been able to describe such behaviours based on QDT set up. The main difference between standard decision theory model and QDT in this case lies in the fact that the formula for total probability (FTP) is different here, there is an additional perturbation term which modifies the FTP such that the impact of contextuality is accounted for (Haven and Khrennikov

2009)[7]. Hence such modifications can capture in probabilistic sense the deviant behaviour of agents under novel contexts such as uncertainty.

QUANTUM LIKE ORDER EFFECTS IN HUMAN COGNITION

The observation is quite simple here, the response to questions asked to agents differ when asked in different orders, however the explanation is not simple at all. Busemeyer et al (2012) have pioneered the use of quantum like modelling set up to explain up to a great extent such order effects in cognition. If the questions are represented by random variables which have positive operator representations (as mentioned earlier, in standard quantum theory states are represented by rays or density matrices in complex Hilbert space, and observables are represented by either self-adjoint projection operators, or more generally positive semi definite operators in the same Hilbert space) then it is not necessary that such operators will commute, or in other words change of orders in operators do produce different output states. Hence if the questions are represented by such non-commuting operators, it is not difficult to see how the final output states or responses by agents will differ, if order of questioning changes.

However, there are still challenges for example what if the questions are repeated? Say in the sequence ABA? Will there be still any order effect? (Aerts et al 2016a) This is still an open question and more general modelling is required (Aerts et al 2016b).

CONJUCTION AND DISJUNCTION EFFECTS

Based on the probabilistic behavioural models, we find at times regular violations of standard probability axioms, for example P(A&B)> P(A)+P(B) the so-called conjunction fallacy, or the opposite of it so called disjunction fallacy, A and B being two events. Certainly, the standard Kolgomorovian measure theory does not accommodate such violations. However, if belief states are described by superposition of basis states in a complex Hilbert space, and measurements are represented by projections on to specific Eigen sub spaces, and probabilities of actualising one final state is given by Born's rule as in quantum theory, then such probability inequalities can be justified. Sequential choices can be described by sequential measures/ projections.

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[7] Such additional terms have been suggested since the seminal works of Neumann and Birkhov (Haven et al 2017), Feynman , recently too there have been multiple versions of this modified FTP (Aerts etal, op cit).

HISENBERG ROBERTSON TYPE INEQUALITIES IN COGNITION/FINANCE

Recently, (Pothos et al 2018) Heisenberg uncertainty relation in form of Robertson inequality has been used to quantify uncertainty in human decision making. As mentioned above conceptualising or describing uncertainty in economic science has always been a challenge, for example often risky situations are used as a description of uncertainty which is inadequate. Mathematical psychologists (Pothos and Busemeyer 2013) have used Hermitian operator representations of in-compatible questions asked to respondents, for example, respondents of elections polls, and have formalised the mutual uncertainties of in-compatible questions to describe the responses. Again, in such models, mental states are dependent on the mutual uncertainties of in-compatible questions[8].

To recollect, cognitive modelling provides a Hilbert space representation of mental or belief states, where the mental state is considered as to be a normalised vector in the state space, or a more general density matrix representation is given which is a mixture or ensemble of the different pure states. Hence it becomes logically possible to use Heisenberg uncertainty principle over such state space description.

Generally, Robertson inequality for any two in-compatible observables, A and B is given by $\sigma(A)\sigma(B) \geq |< [A, B] >/2|$, Where the right-hand side lower limit does generally depend upon the state of the system.[9]. In case of cognitive modelling the state dependence of the lower bound of the uncertainty relation was emphasised by Khrennikov and Haven (2007).

2. CONCLUSION: *OTHER DIMENSIONS OF QUANTUM LIKE MODELLING*

There have been a number of alternative quantum or quantum field theoretic formulation of asset markets in recent years (Havens and Bagarello, Youkolov and Sornette, please check ref list below). These studies focus on modelling the interaction between the traders based on the operator formalism in the quantum

[8]In such models, questions are always represented as operators: positive operators, or at times more restrictedly as self-adjoint projector operators, and the action of such operators are to project the mental states into one of the Eigen subspaces of the initially superposed belief/ mental states in the basis of the Hilbert space concerned.

[9]In Quantum physics the commutator bracket between position and momentum is special since the RHS of the equation is state independent.

field theory. For example, in the earlier presented two-agent model (two agents being Alice and Bob), if we introduce the interaction between the traders and the information environment, or the so-called Bath (which can be considered as the huge bath comprising of many degrees of freedom, comprised of hard and soft information) then a new model of decision-making can be formulated. In such a case, formulation agents may start with some initial pure states of beliefs, however, when they interact with the information environment such states 'decohere' and converts into mixed states. This thinking is based on the decoherence theory of modern quantum mechanics, but again proper interpretations of parameters are warranted. Here, we need to formulate the Hamiltonian of the system which is again comprised of different creation and destruction operators with their commutation relationships, then there can be different conserved quantities represented by the so-called number operators (for example, in a restricted model the total no of shares traded in the market can be conserved, however, these shares are just traded or exchanged from one trader to another), and finally we need to solve the time evolution of such operators which would provide us with the time evolution equations for the market as a whole. There are still many challenges in the road ahead, for example, developing a fruitful model of n interacting agents, where n can be >2 and large.

To conclude we should remember that there are numerous interpretations of the famous 'measurement' problem in QM, i.e., there seem to be two inconsistent processes happening, until we measure the superposed sate of a system evolves according to Schrödinger equation/ the unitary evolution method, but as soon as a measurement is performed we get a so called wave function collapse in a random fashion, these two procedures are inconsistent. There are several interpretations (multiverse theory, pilot wave theory, spontaneous collapse of wave function, decoherence theory etc.), however application of quantum theory in decision making at large may still give rise to equally challenging issues, but a new beginning also.

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